Russia IYPT

Sugar and salt

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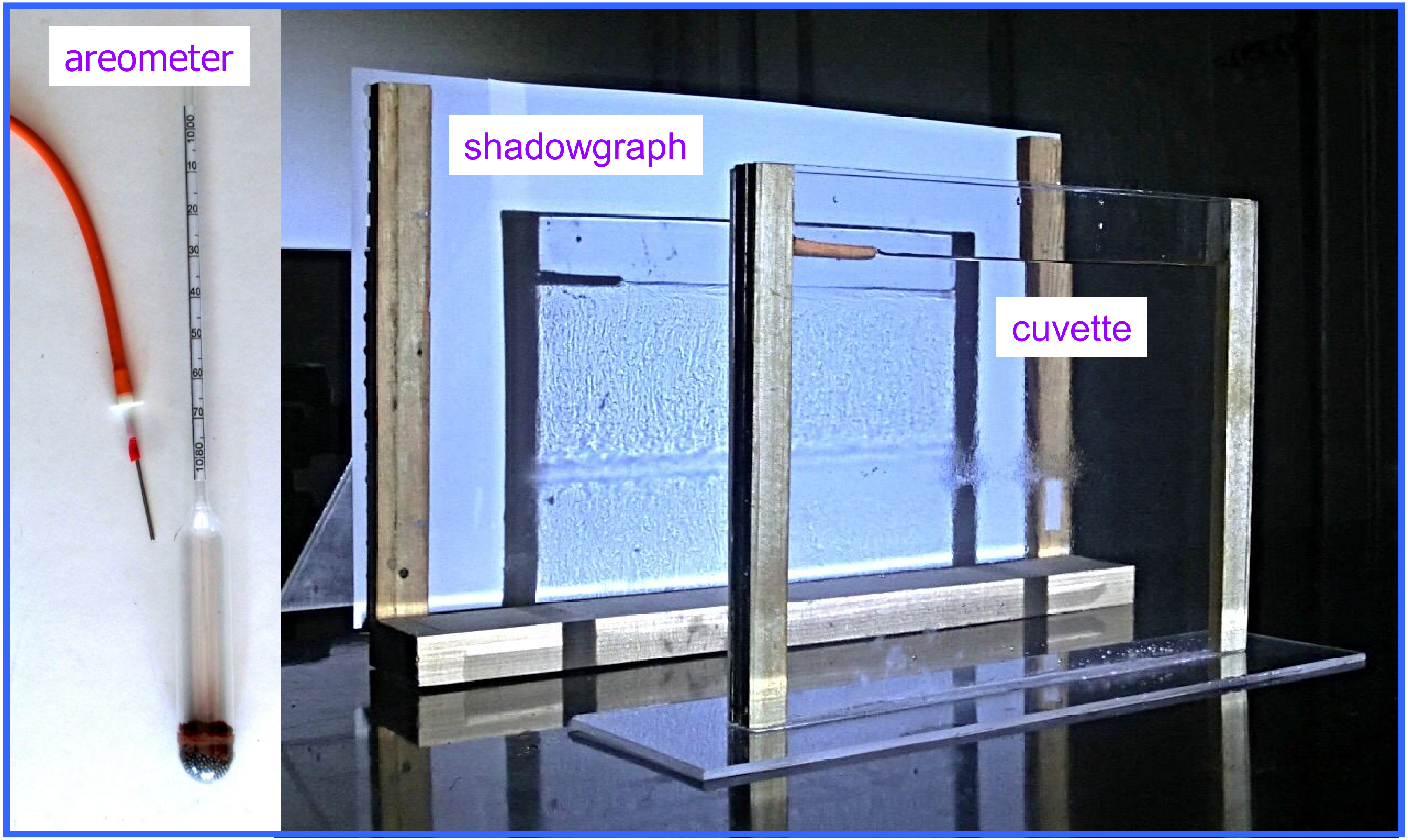
The problem

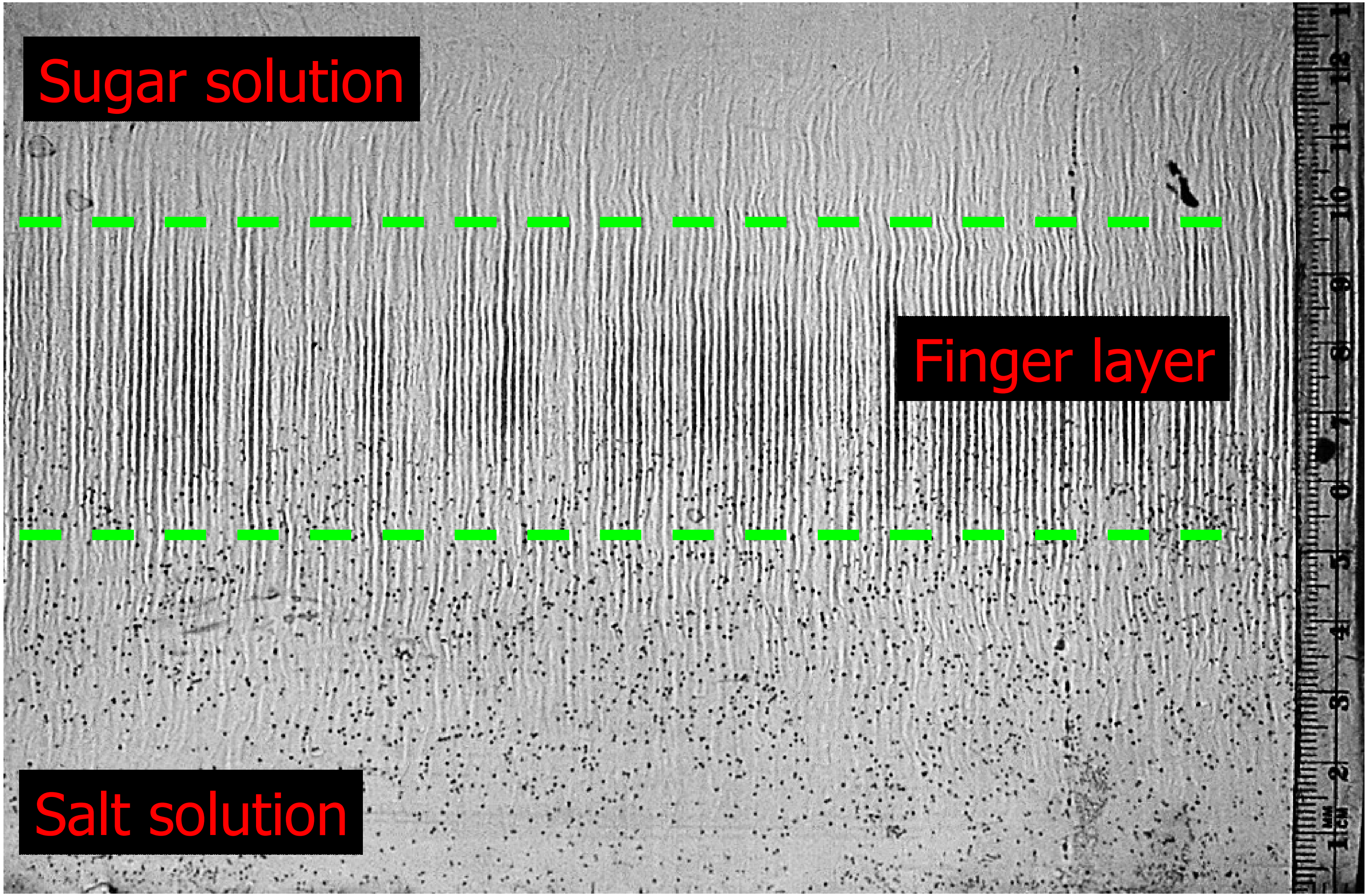
When a container with a layer of sugar water placed above a layer of salt water is illuminated, a distinctive fingering pattern may be seen in the projected shadow. Investigate the phenomenon and its dependence on the relevant parameters.

First observations

Experimental setup



Interpenetrated fingers



Plan of the report

Qualitative explanation

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Soft and hard regimes of fingers growing

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Theoretical model

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Experimental measurements

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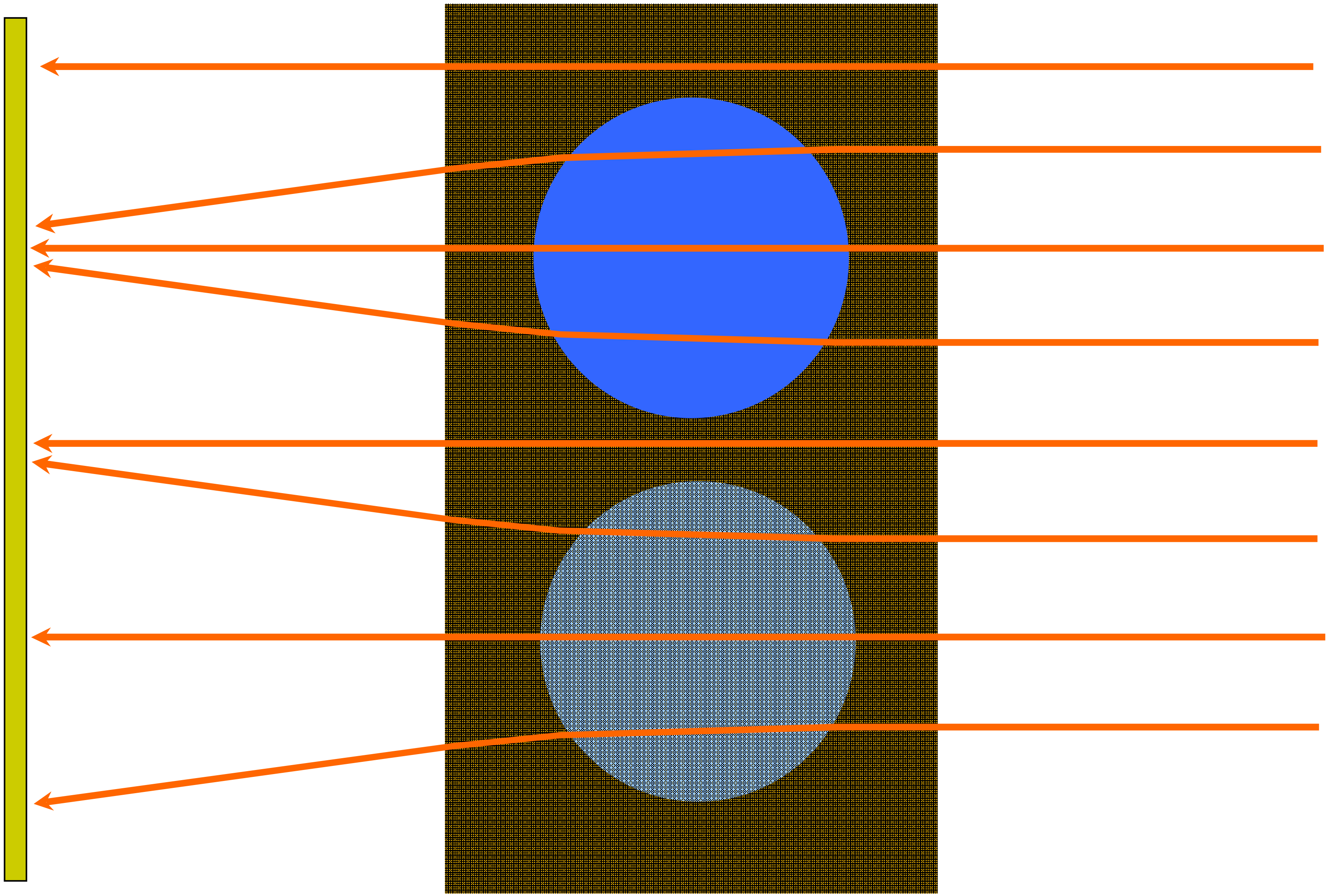
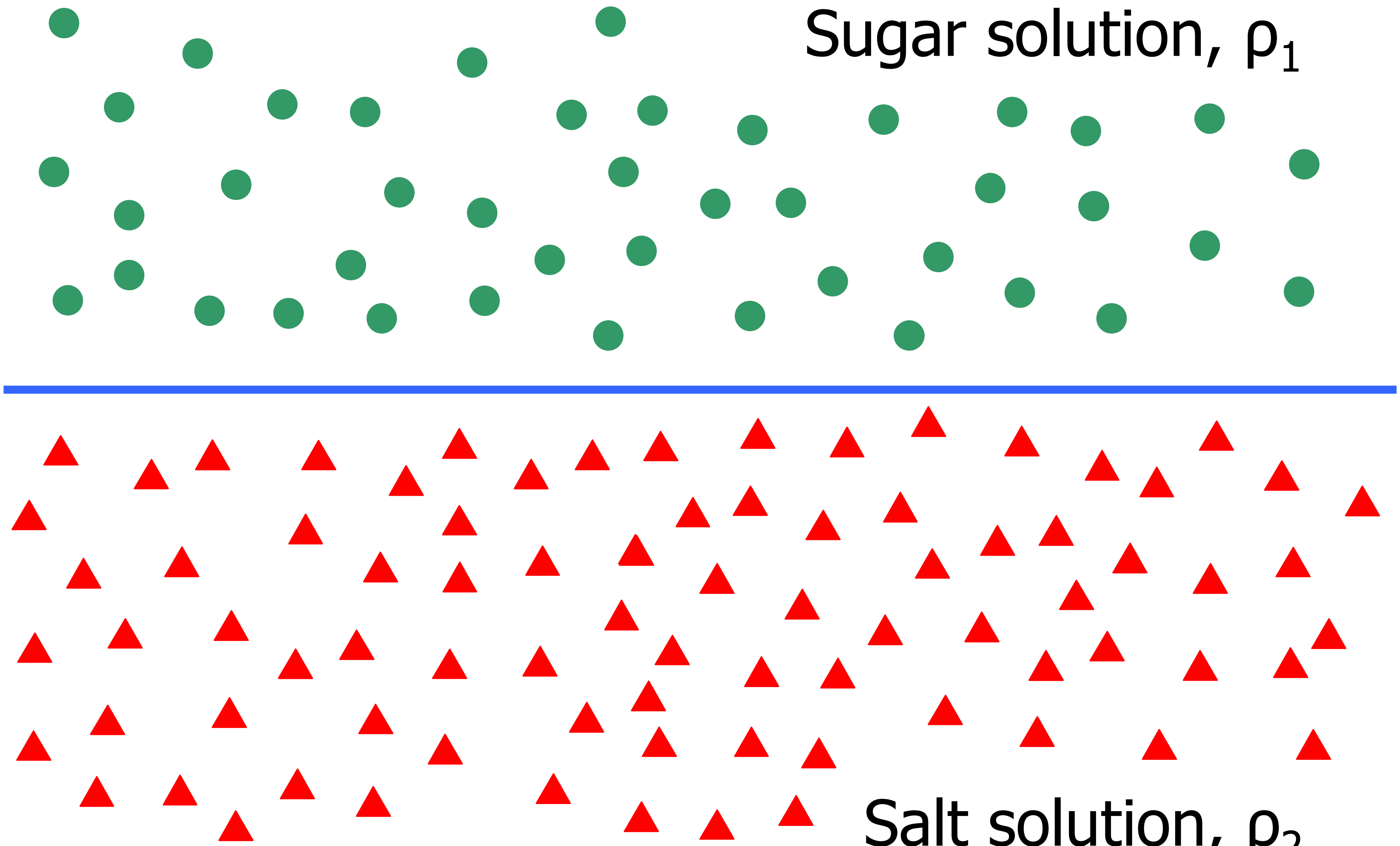
Width of the finger

Qualitative explanation

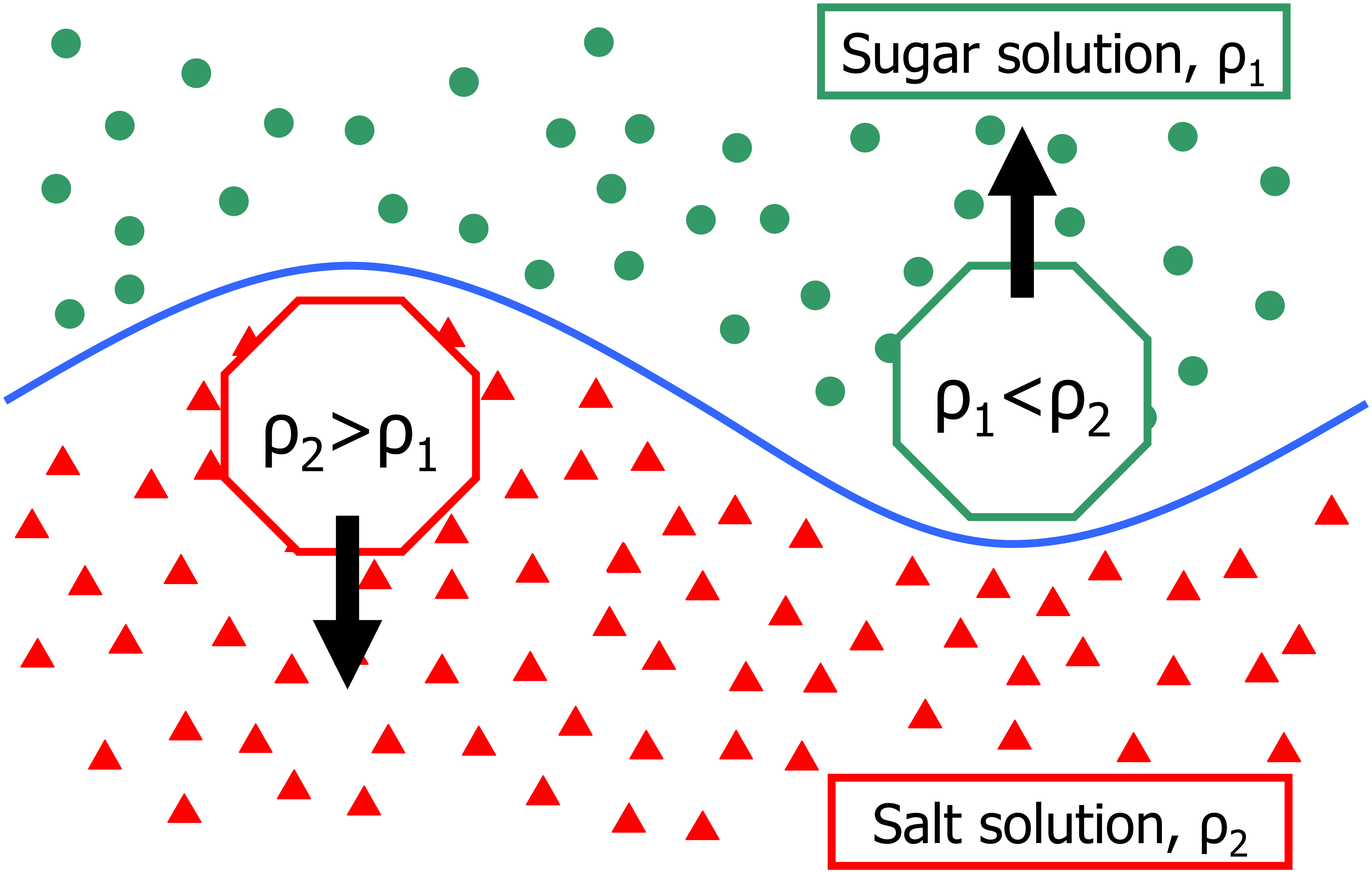
Optical inhomogenities



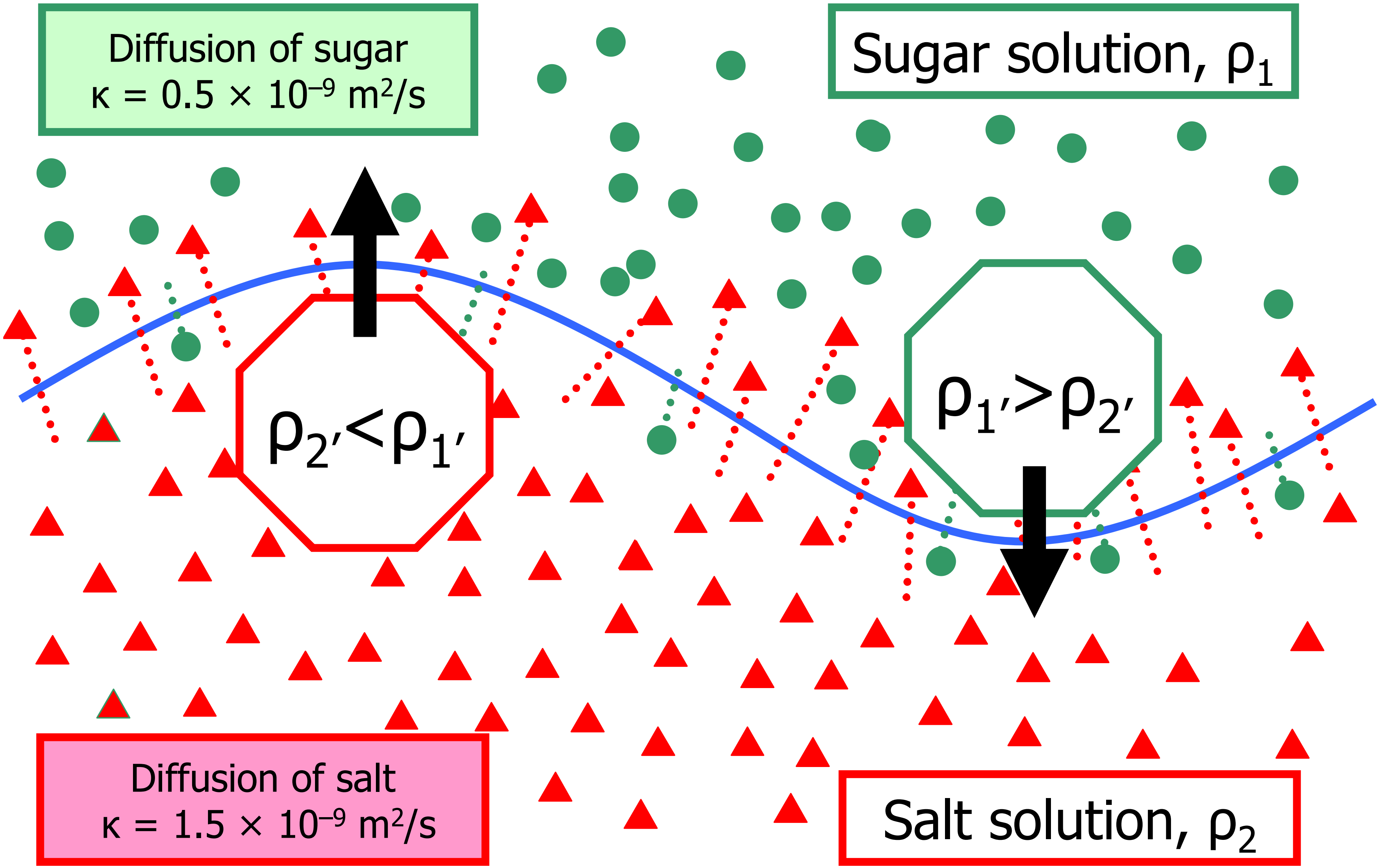
Initial state



Perturbed boundary (stable)



Double diffusion (unstable)



Relevant parameters

- Ratio of densities
- Duration of experiment
- Liquid's viscosity
- Different solvents
- Coefficient of diffusion
- Different solutions

When a container with a layer of <u>sugar water</u> placed above a layer of <u>salt water</u> is illuminated, a distinctive fingering pattern may be seen in the projected shadow.

Density ratio

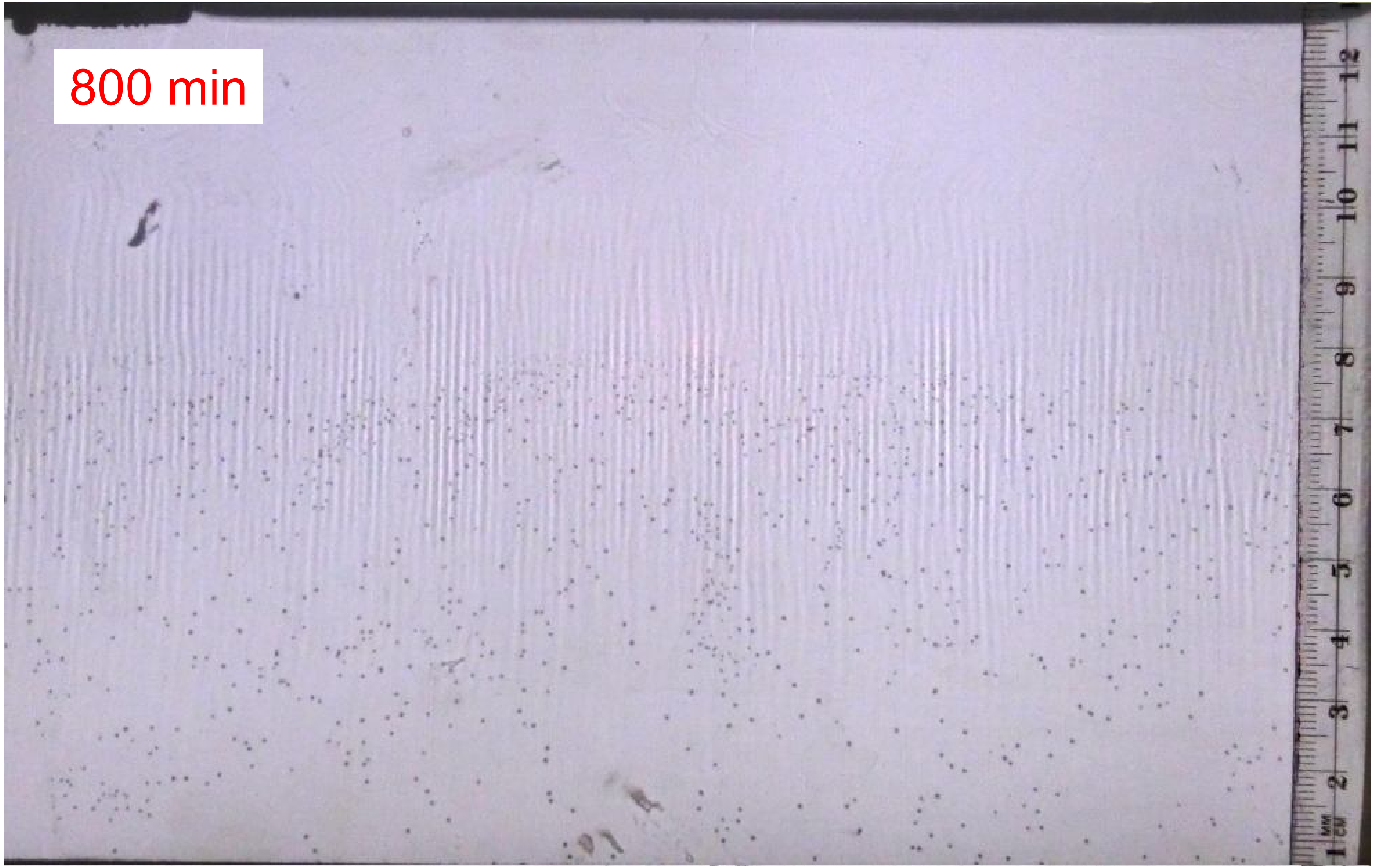
Sugar solution, $\rho_1 = \rho_{water} + \varepsilon_1$

$$Q = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\rho_1 - \rho_{water}}{\rho_2 - \rho_{water}}$$

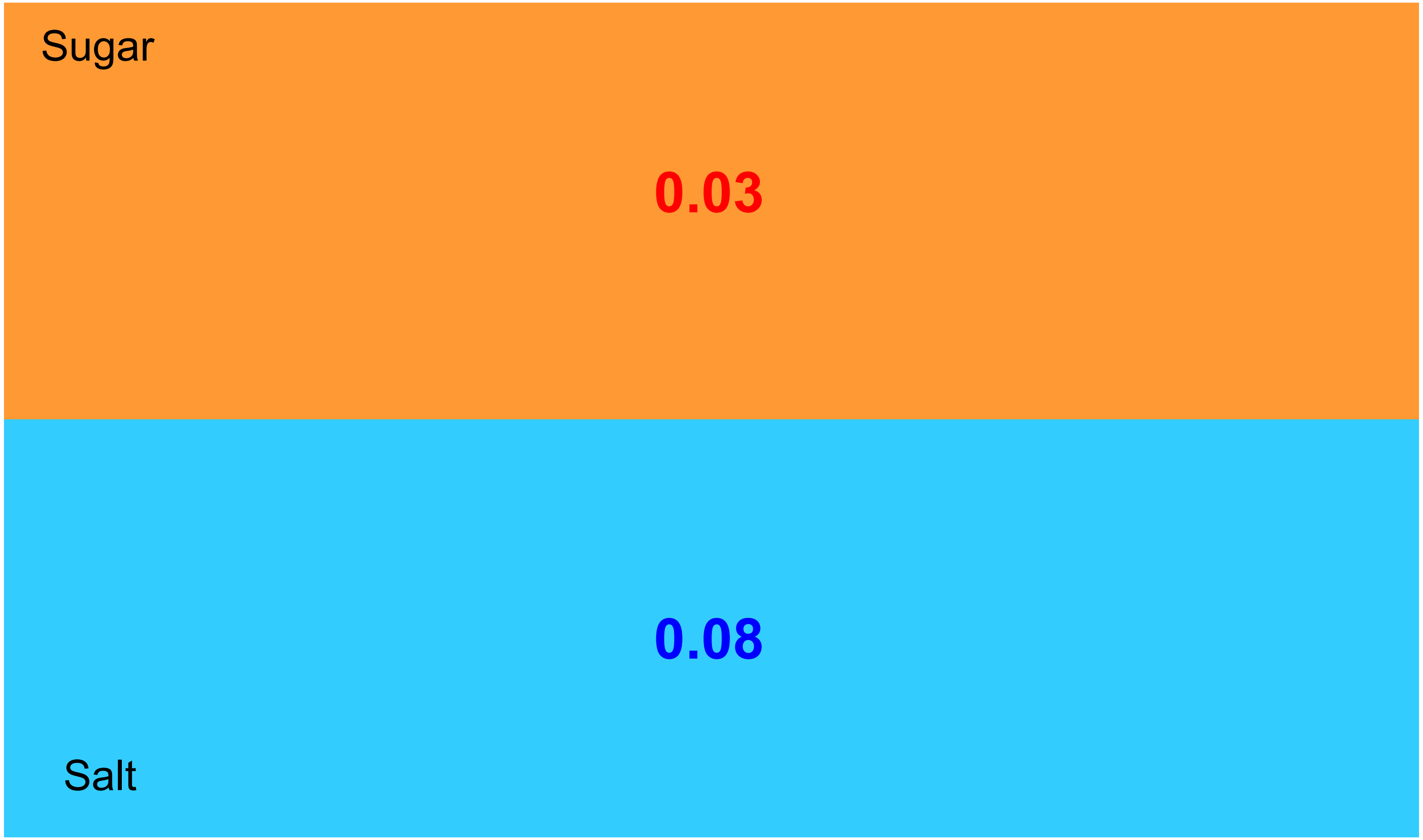
Salt solution, $\rho_2 = \rho_{water} + \varepsilon_2$

$Q = 0.37$:
soft regime

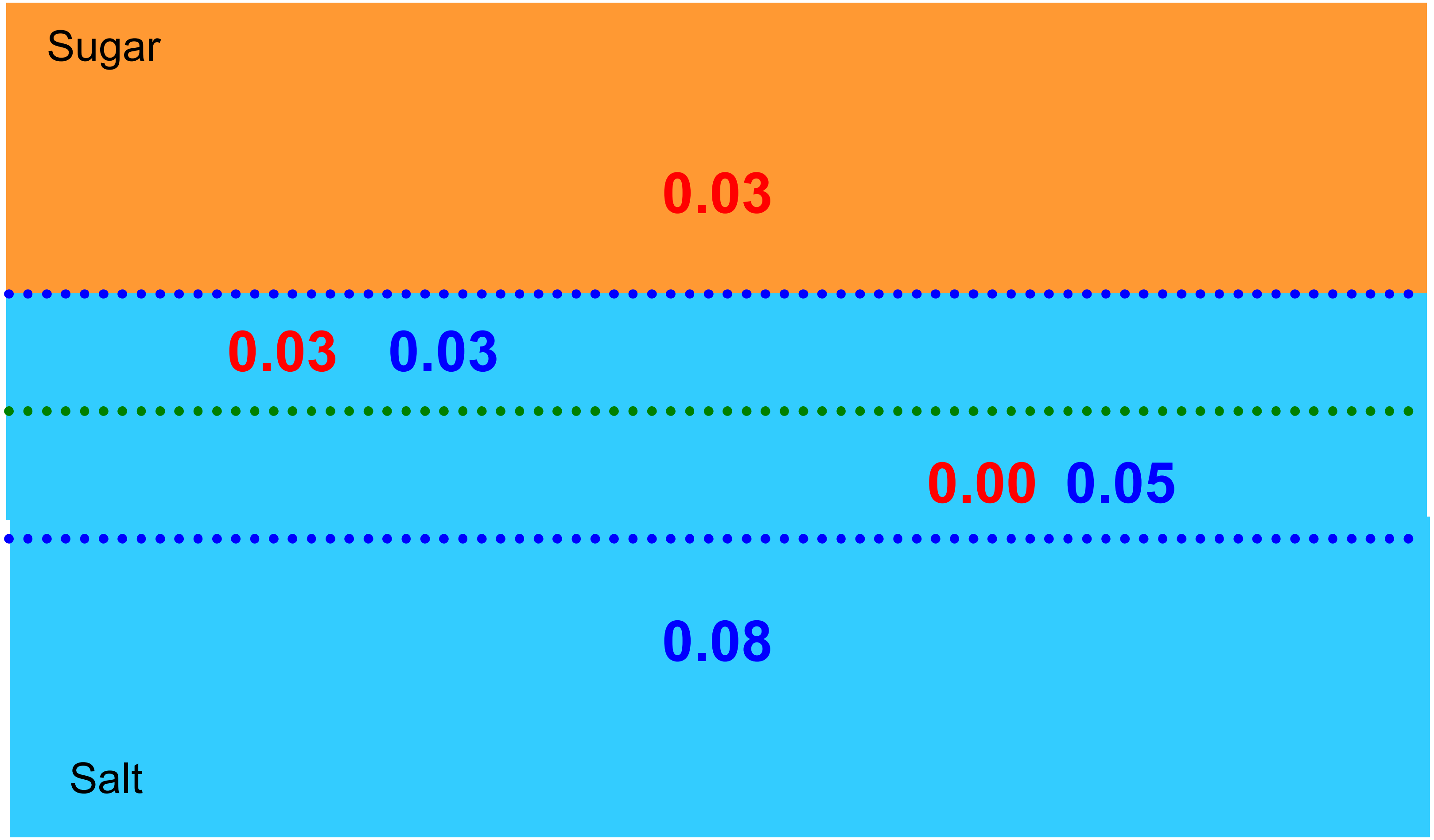
Sugar 1.03 salt 1.08   Q = 0.37



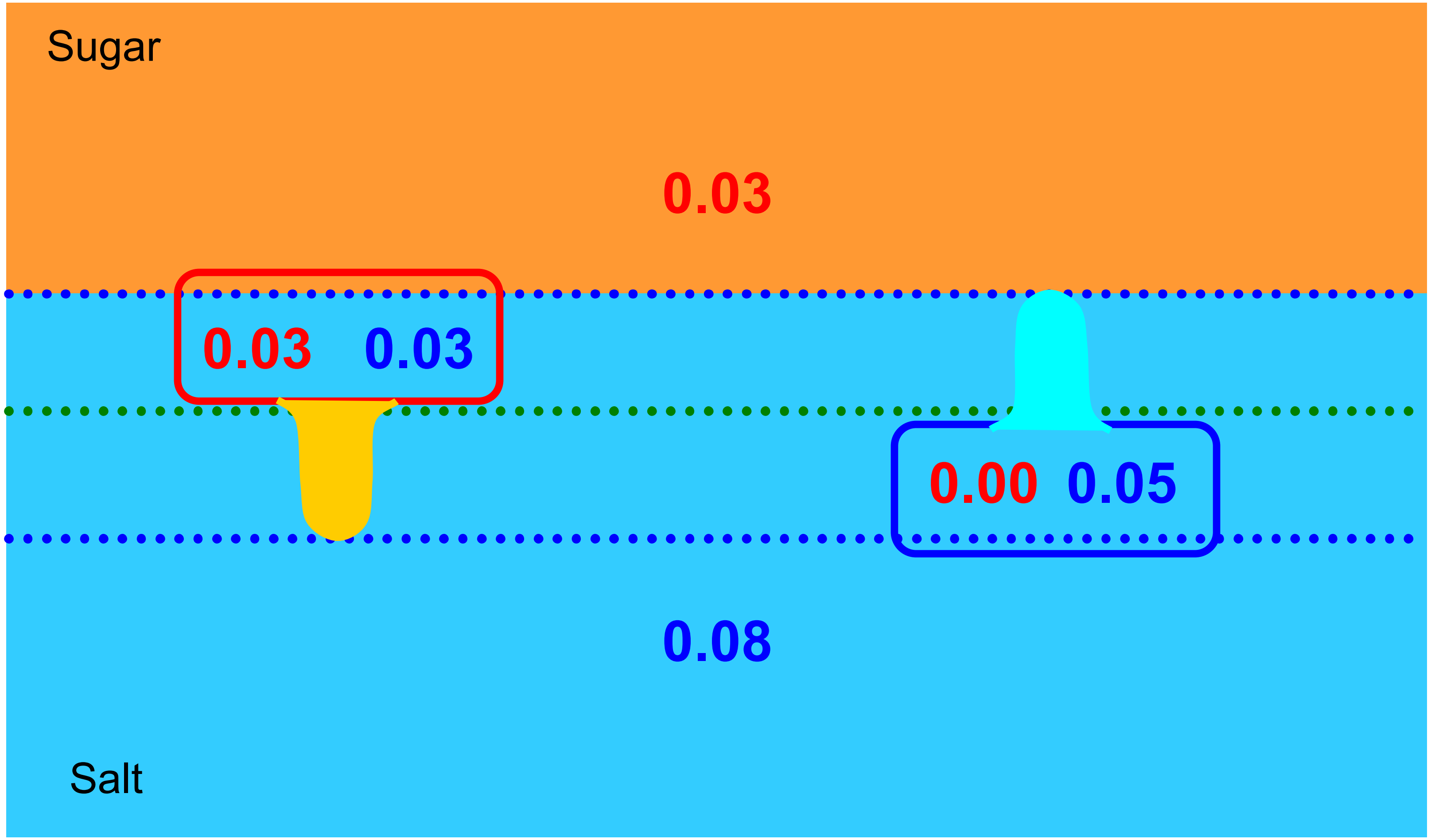
Initial condition (Q = 0.37)



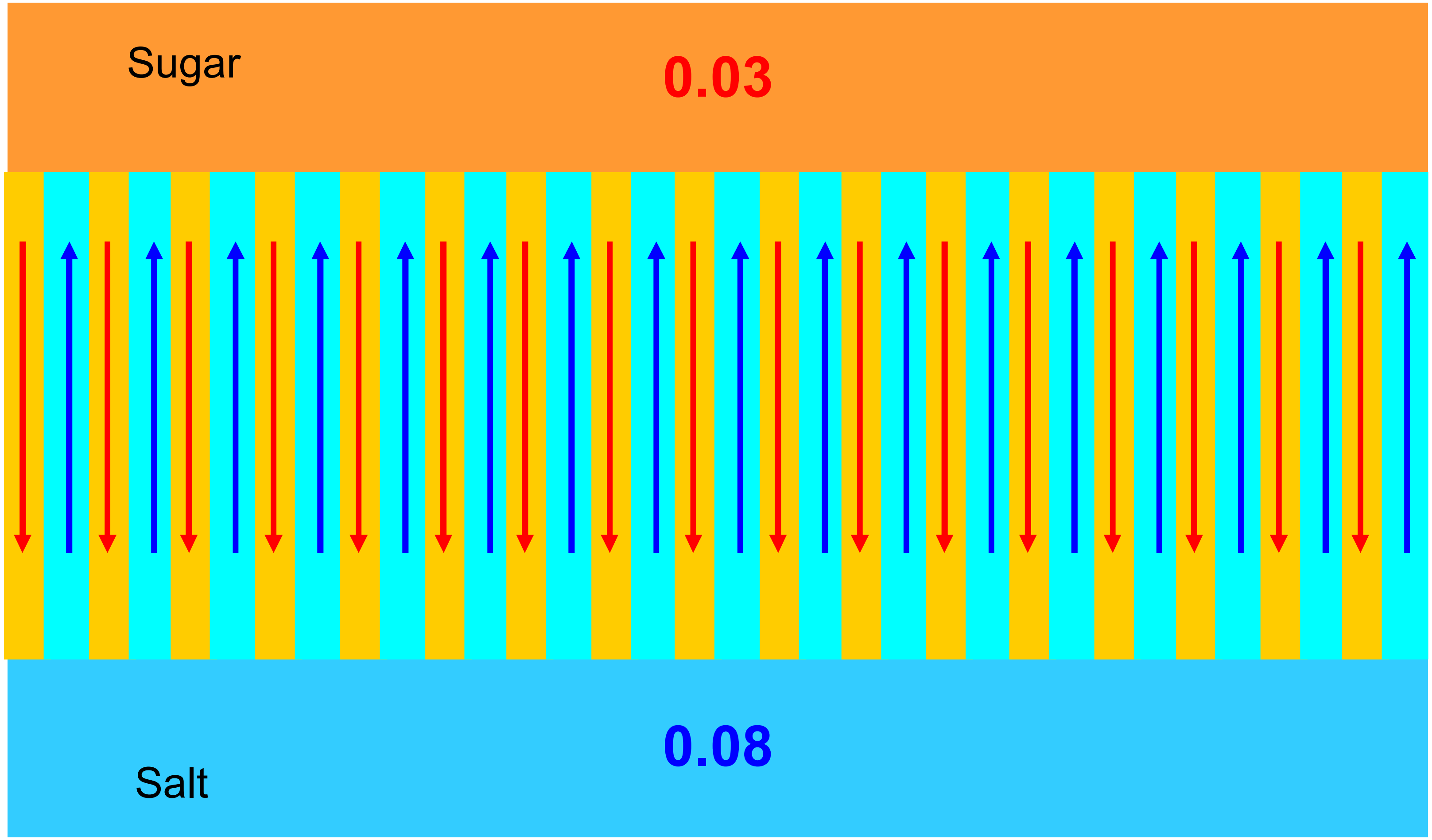
Double diffusion



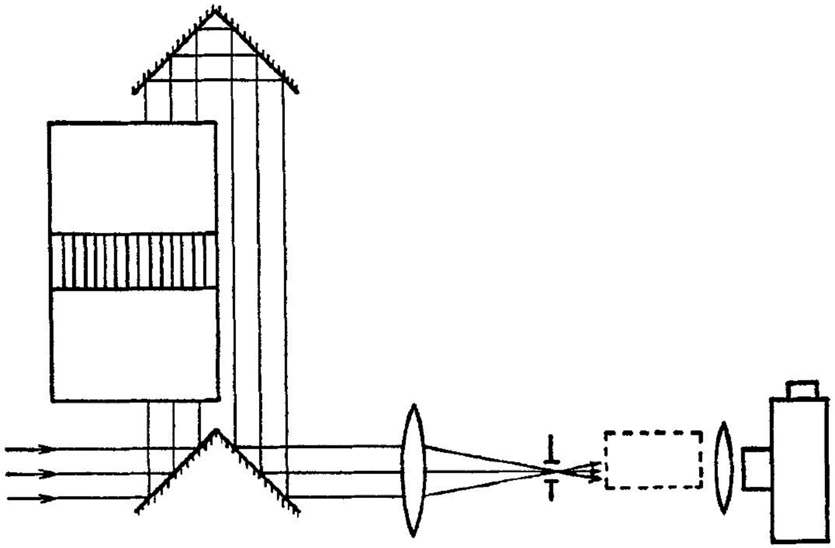
Rayleigh-Taylor instability

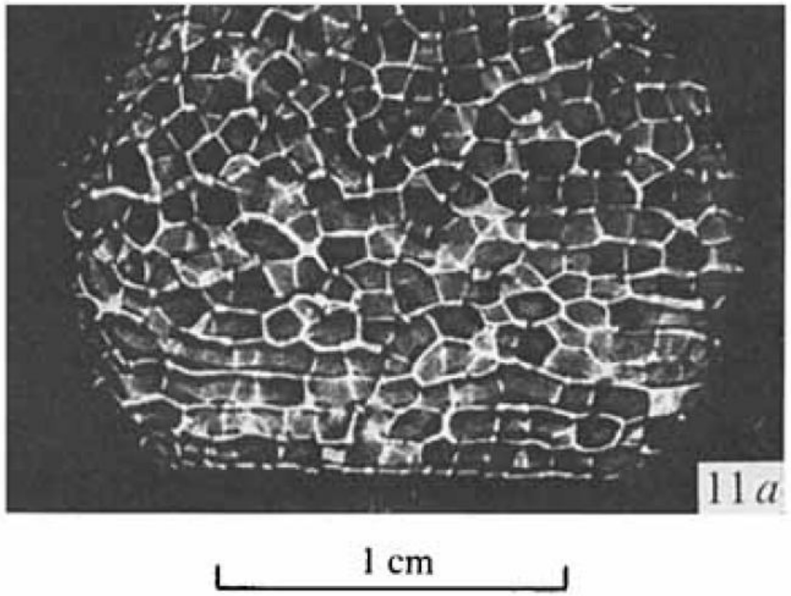


Dissipative self-organization



Fingers as they look from above

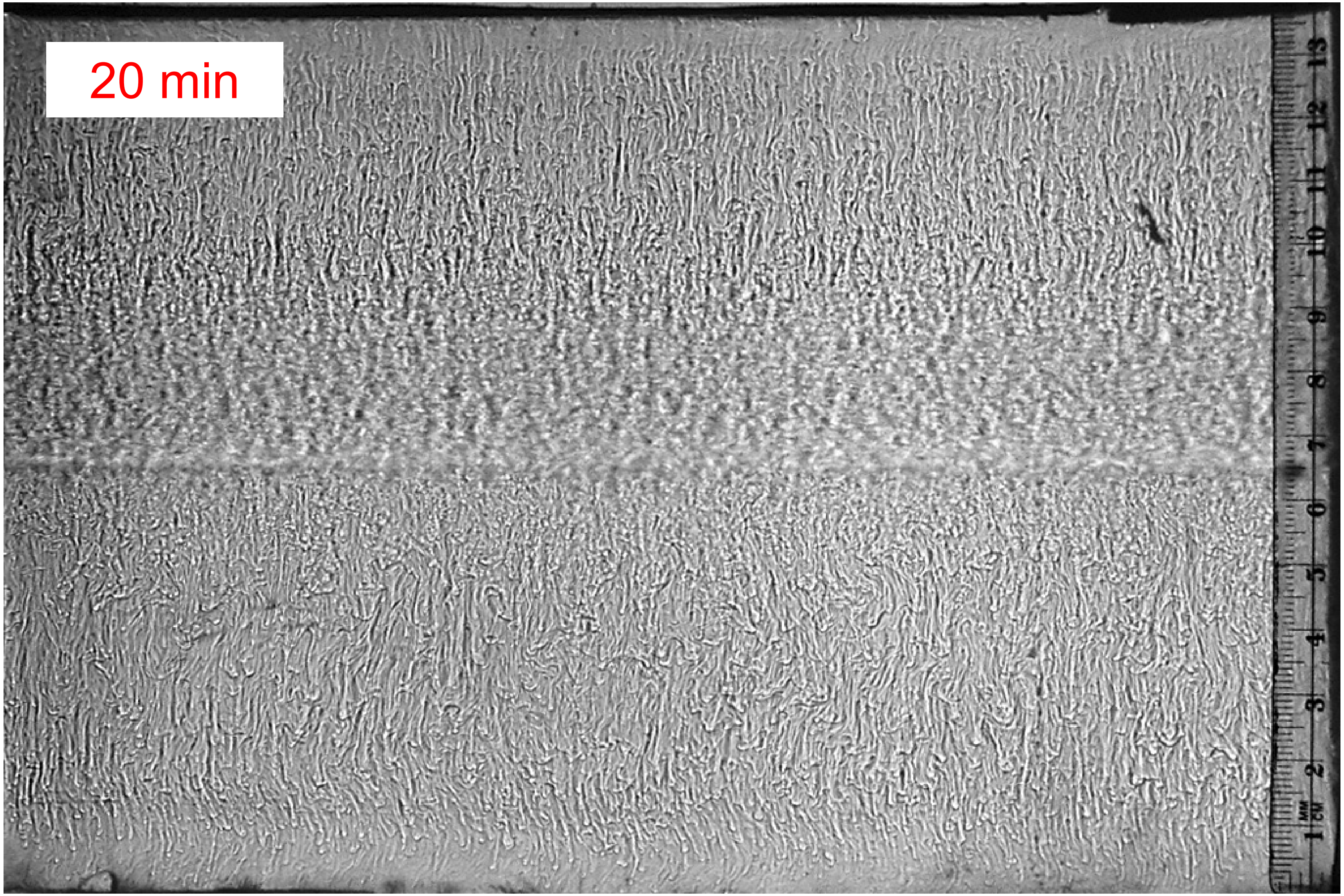




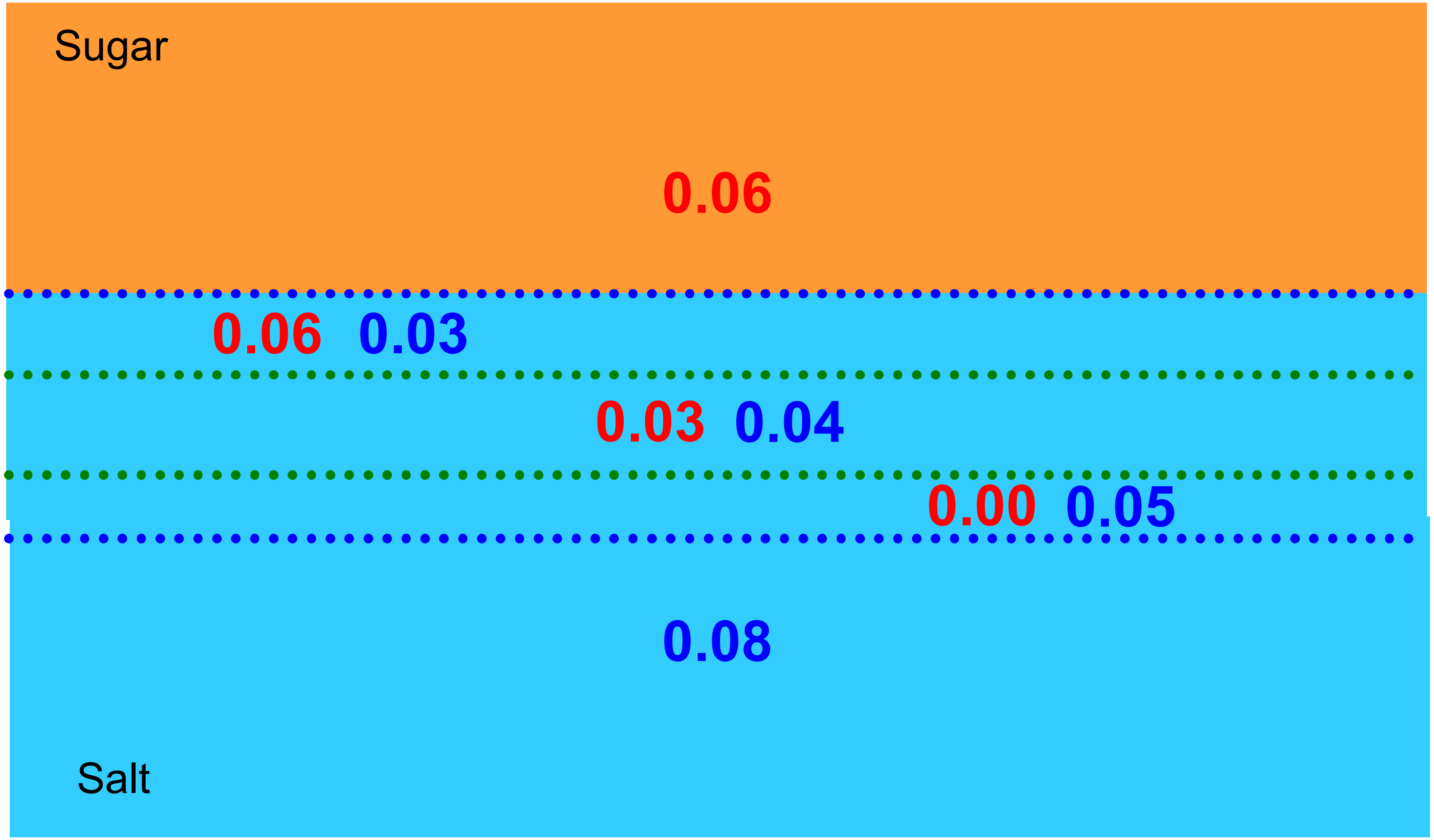
Shirtcliffe T.G.L., Turner J.S. (1970) “Observations of the cell structure of salt fingers”.

$Q = 0.75$:
hard regime

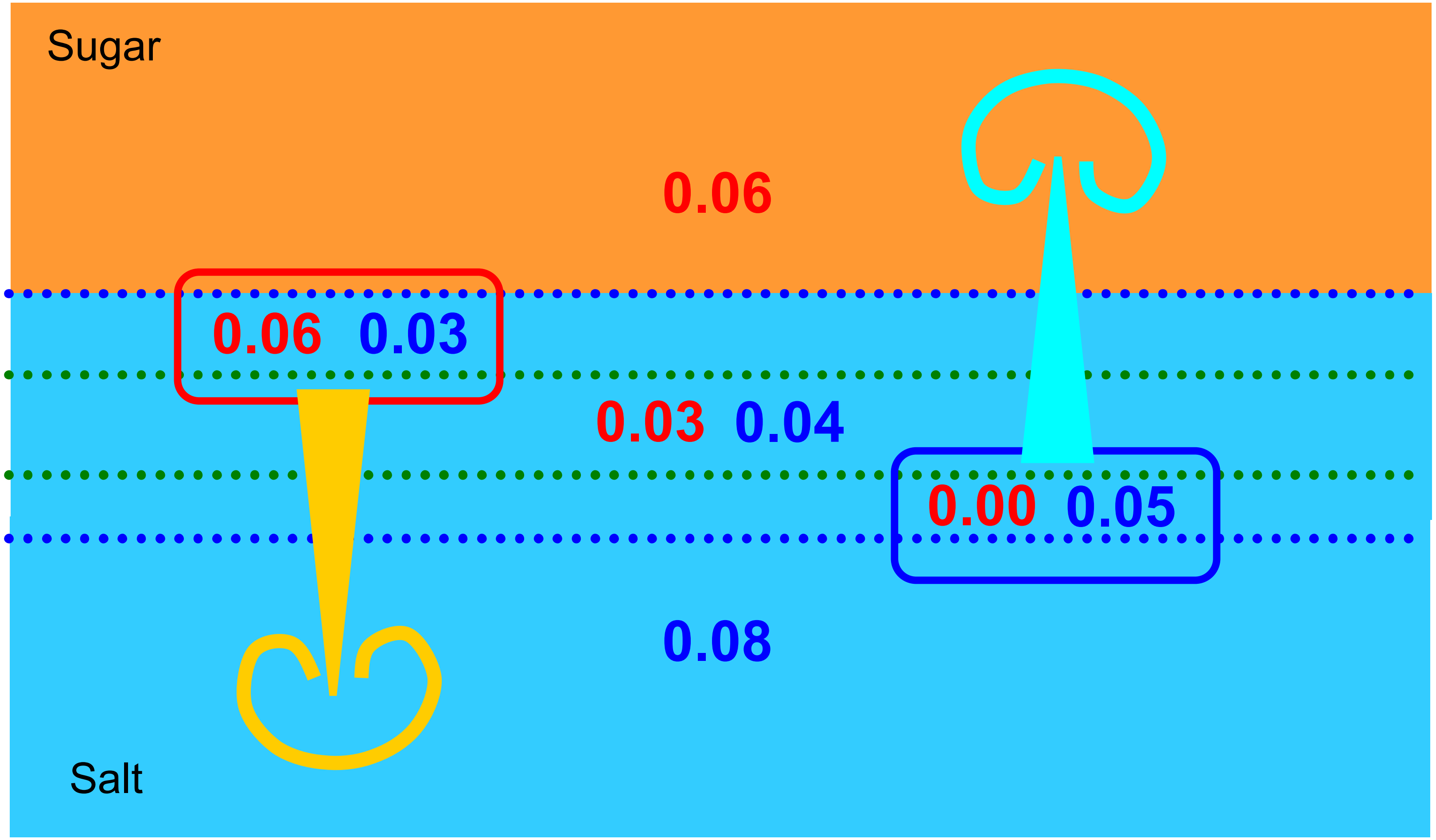
Sugar 1.06 salt 1.08 Q = 0.75



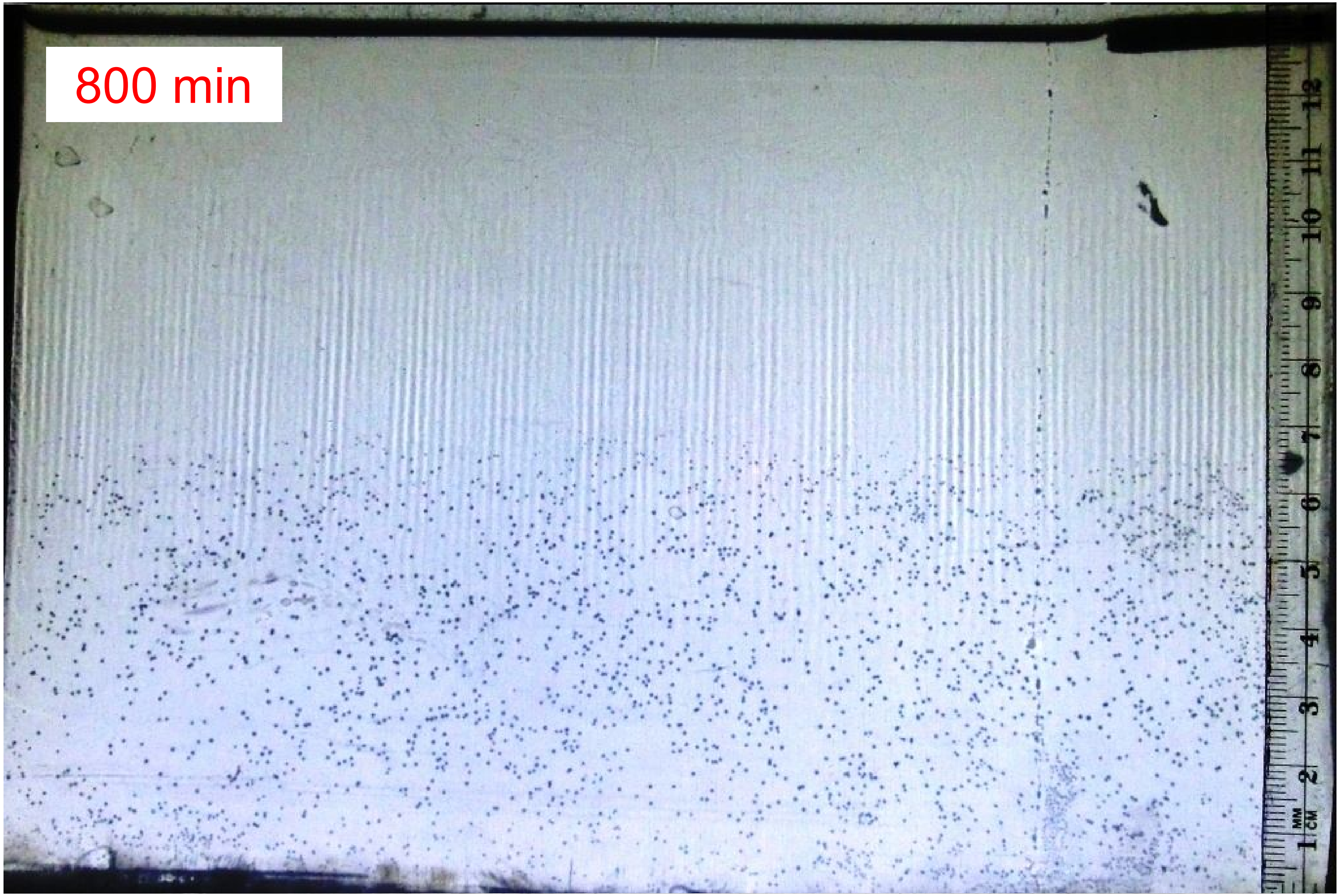
Diffusion of salt and sugar



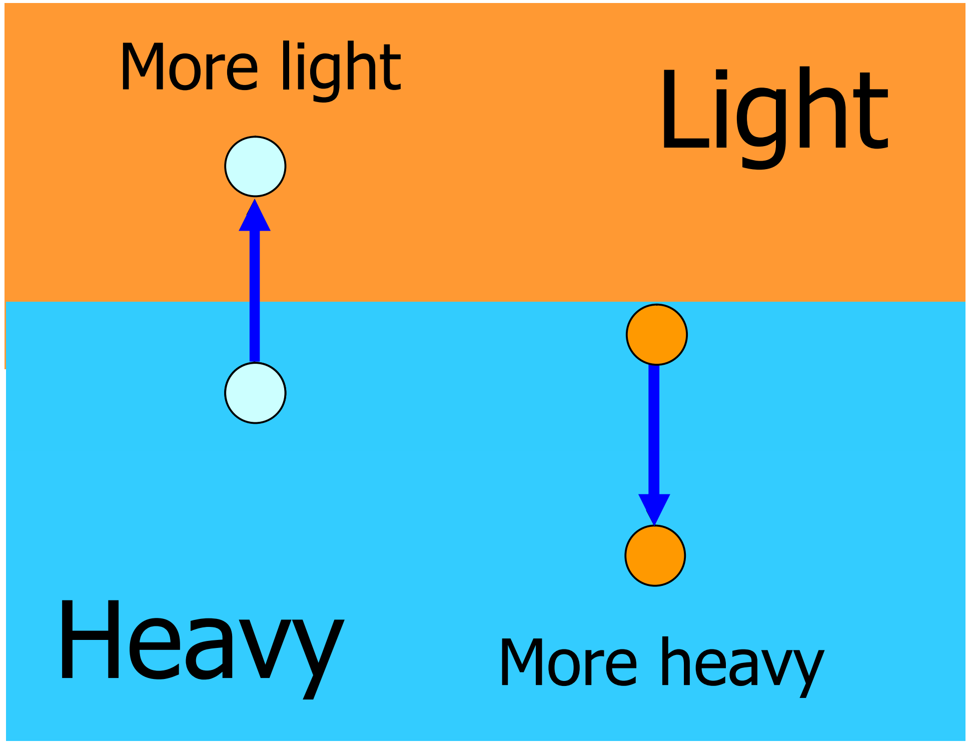
Rayleigh-Taylor instability



Sugar 1.06 salt 1.08   Q = 0.75



Paradox of self-organization

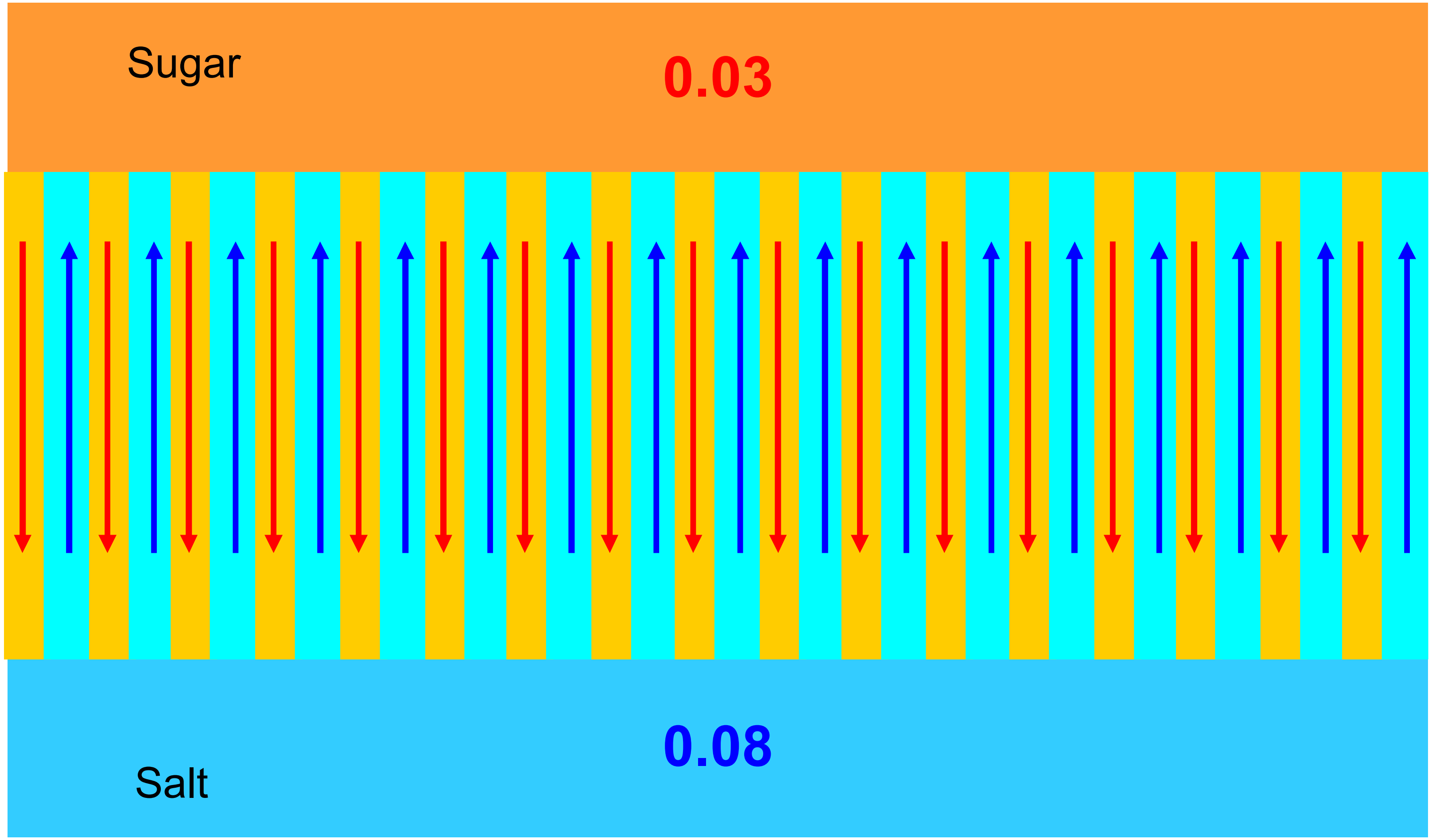


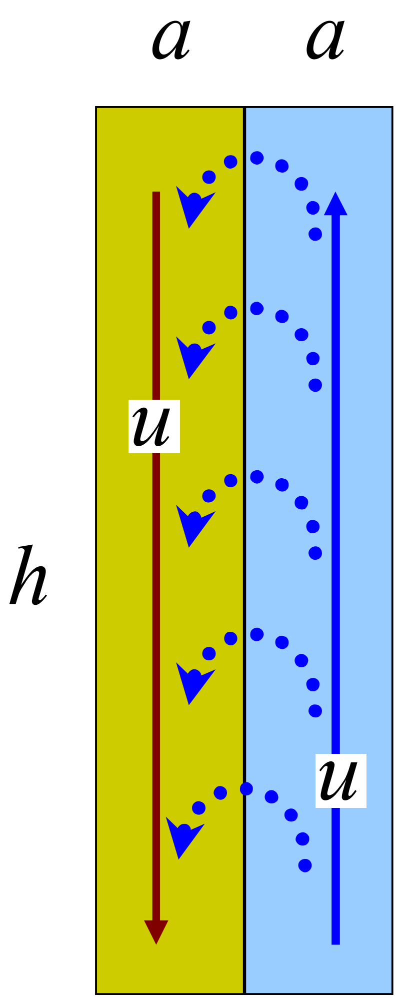
Inhomogeneity in density increases?!

Inhomogeneity of concentration is equalized!

Theoretical model

Dissipative self-organization





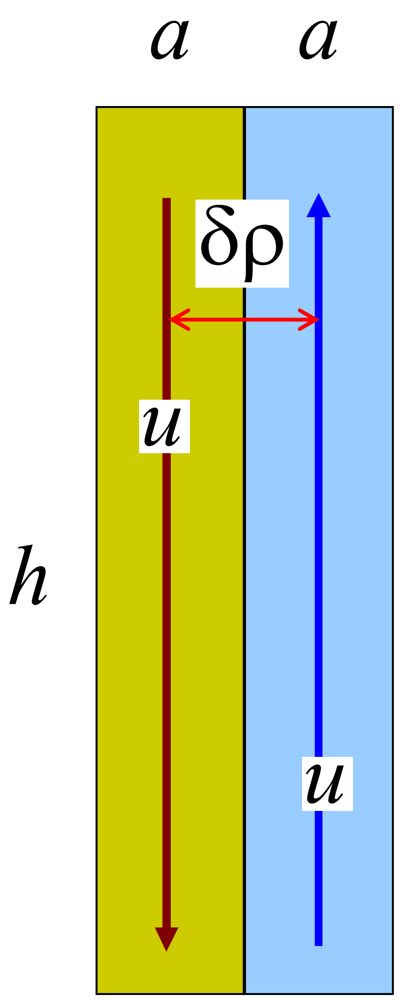
$$\kappa = 1.5 \cdot 10^{-9}\,\mathrm{m}^2/\mathrm{s}$$

Convective flux $\rho_S u \cdot a^2$

Diffusive flux $\kappa \cdot \dfrac{\rho_S}{a} \cdot ah$

Flux balance $u \simeq \dfrac{\kappa h}{a^2}$

Density difference



$$\nu = 0.9 \cdot 10^{-6}\,\mathrm{m}^2/\mathrm{s}$$

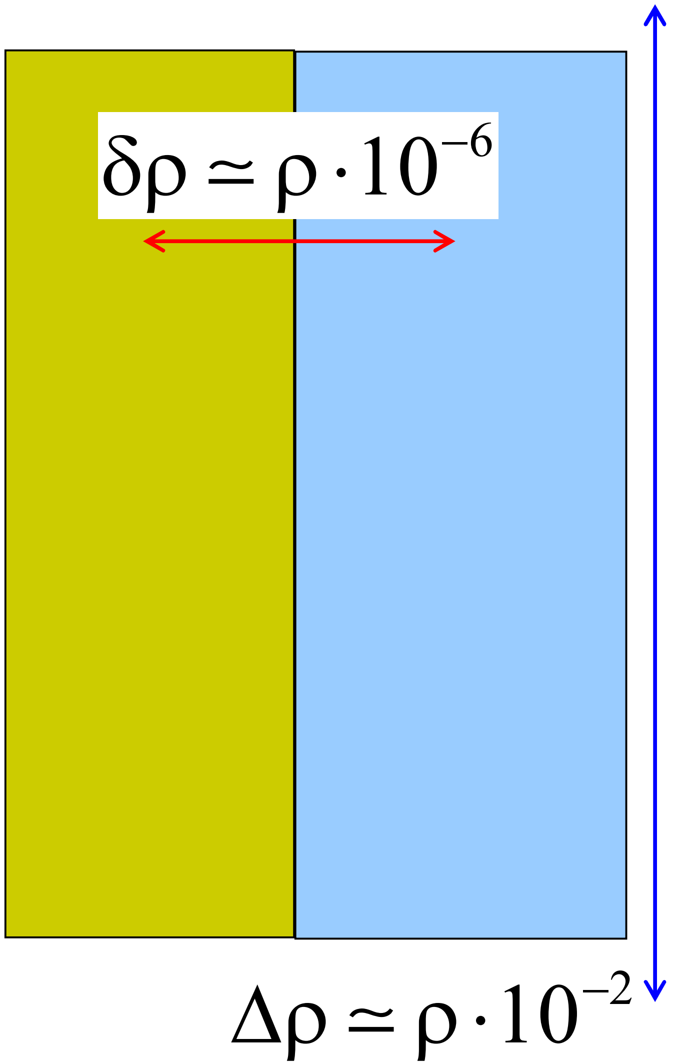
Buoyancy force $\delta\rho \cdot a^2 h \cdot g$

Viscous force $\nu\rho \cdot \dfrac{u}{a} \cdot ah$

Force balance $\dfrac{\delta\rho}{\rho} \simeq \dfrac{\nu u}{a^2 g}$

$$\boxed{\frac{\delta\rho}{\rho} \simeq \frac{\kappa\nu}{g} \cdot \frac{h}{a^4} \simeq 10^{-6}}$$

Density difference



$$\frac{\delta\rho}{\Delta\rho} \simeq 10^{-4}$$

$$\frac{a}{h} \simeq 10^{-2}$$

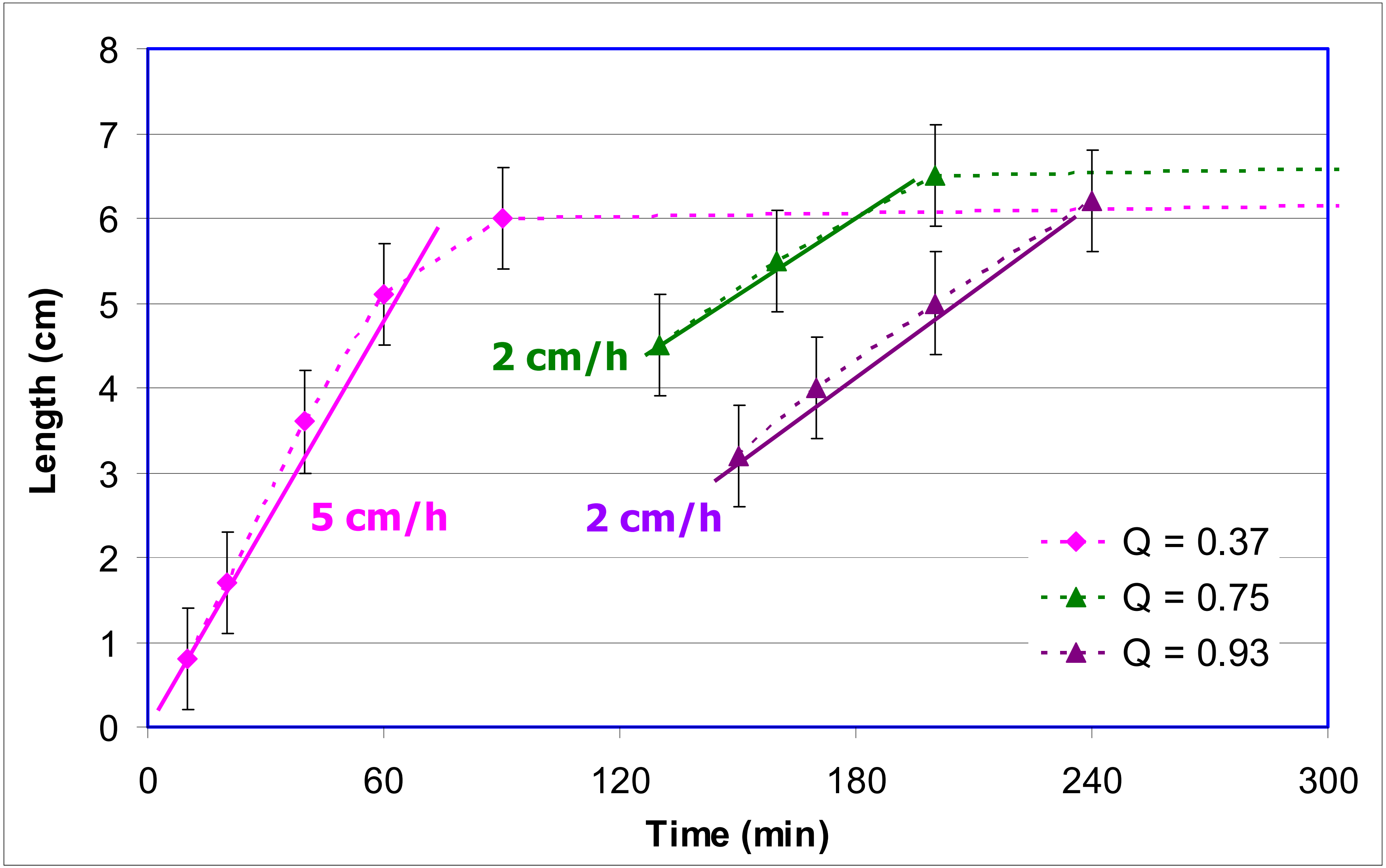
$$\frac{\delta\rho}{\Delta\rho} \sim \left(\frac{a}{h}\right)^2$$

Velocity in fingers

$$\frac{\delta\rho}{\rho} \simeq \frac{\kappa\nu}{g}\cdot\frac{h}{a^4} \qquad u \simeq \frac{\kappa h}{a^2} \qquad \frac{\delta\rho}{\Delta\rho} \sim \left(\frac{a}{h}\right)^2$$

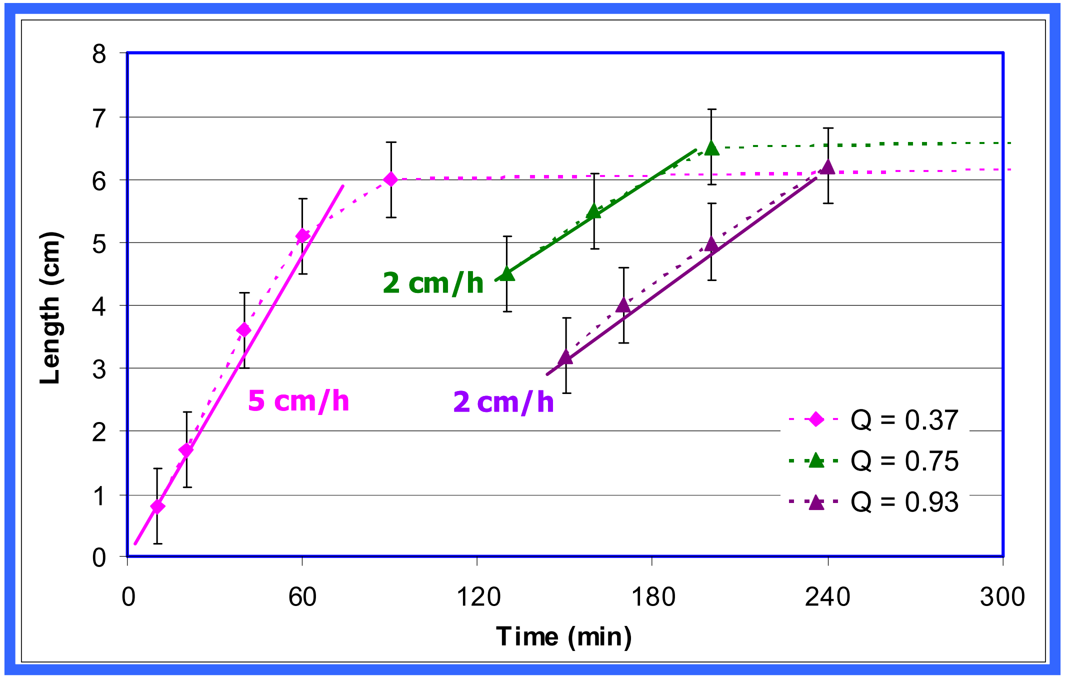
$$u \sim \left(\frac{\Delta\rho}{\rho}\cdot\frac{\kappa^2 g}{\nu}\right)^{1/3}$$

Increase in length



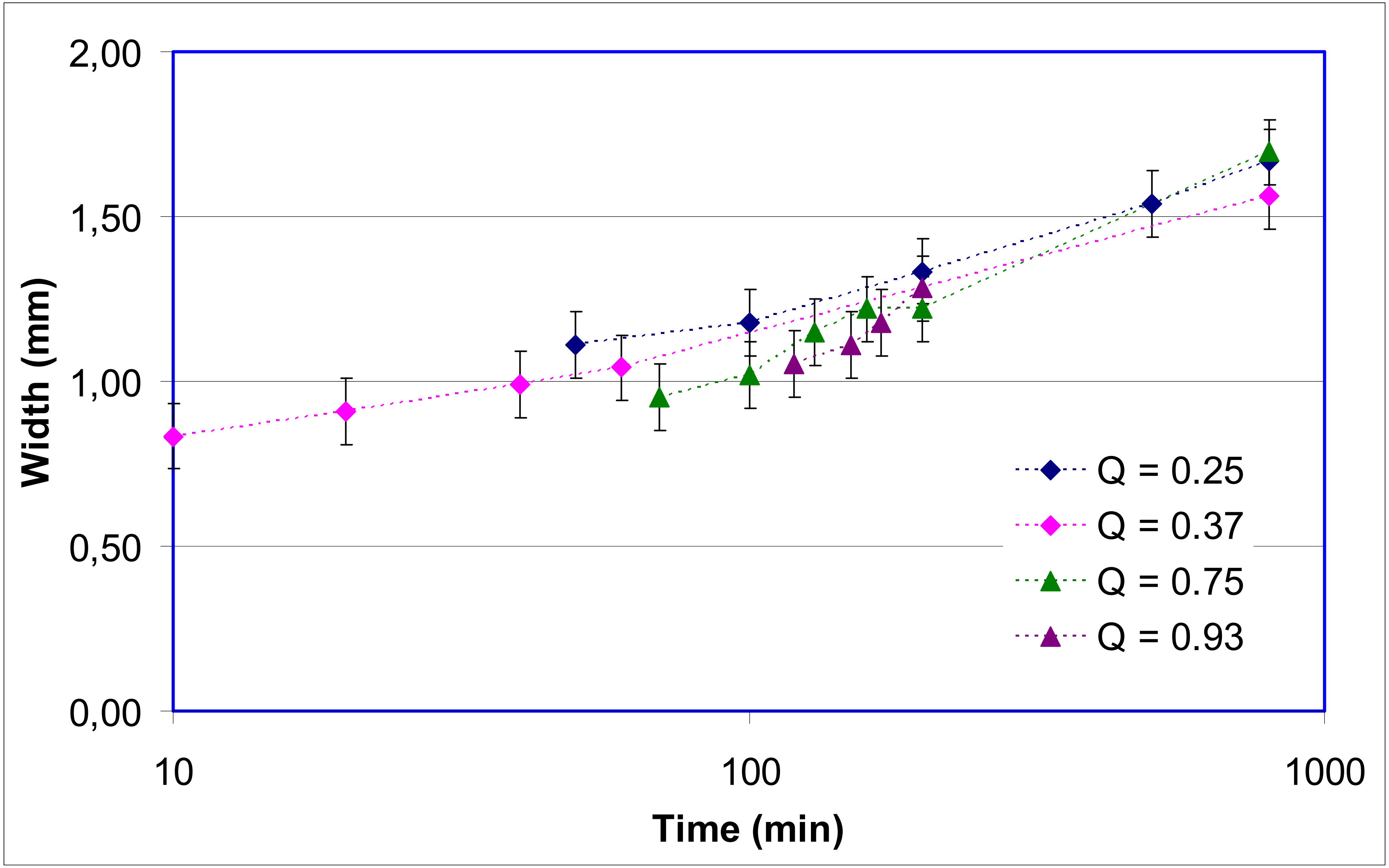
$$u \sim \left(\frac{\Delta\rho}{\rho} \cdot \frac{\kappa^2 g}{\nu} \right)^{1/3}$$

$$\boxed{u \sim 2 \text{ cm/h}}$$



Width of fingers

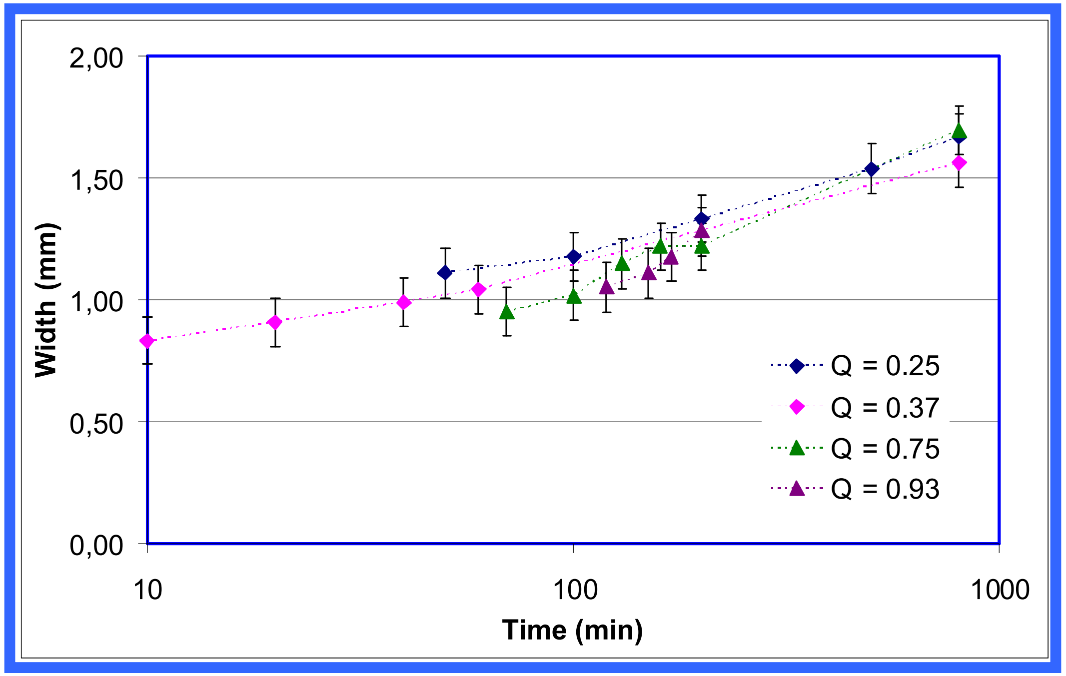
Increase in width



Width of fingers

$$a \sim \left(\frac{\rho}{\Delta\rho} \cdot \frac{\kappa\nu}{g} \right)^{1/6} \cdot h^{1/2}$$

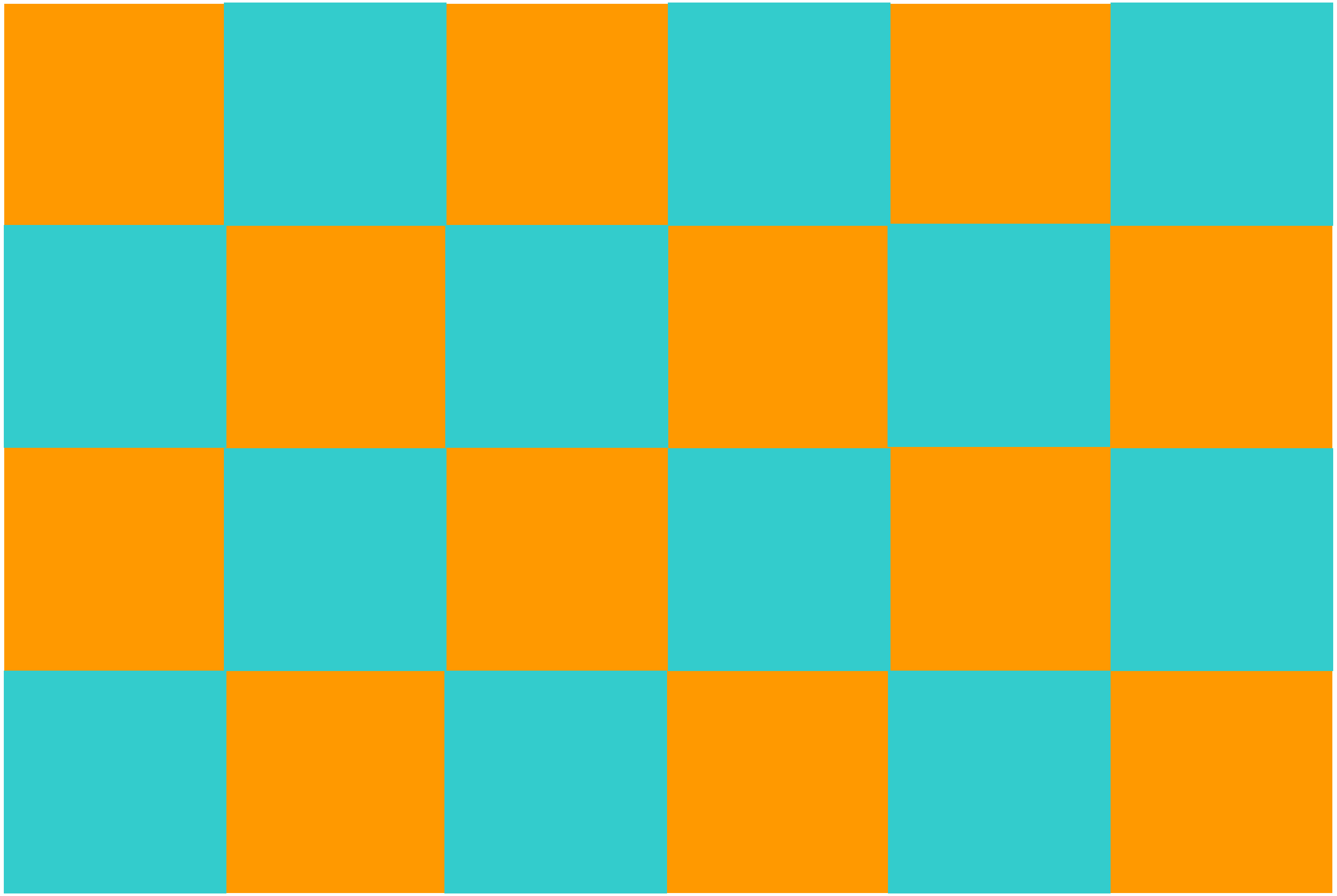
$$\boxed{a \sim 1\ \mathrm{mm}}$$



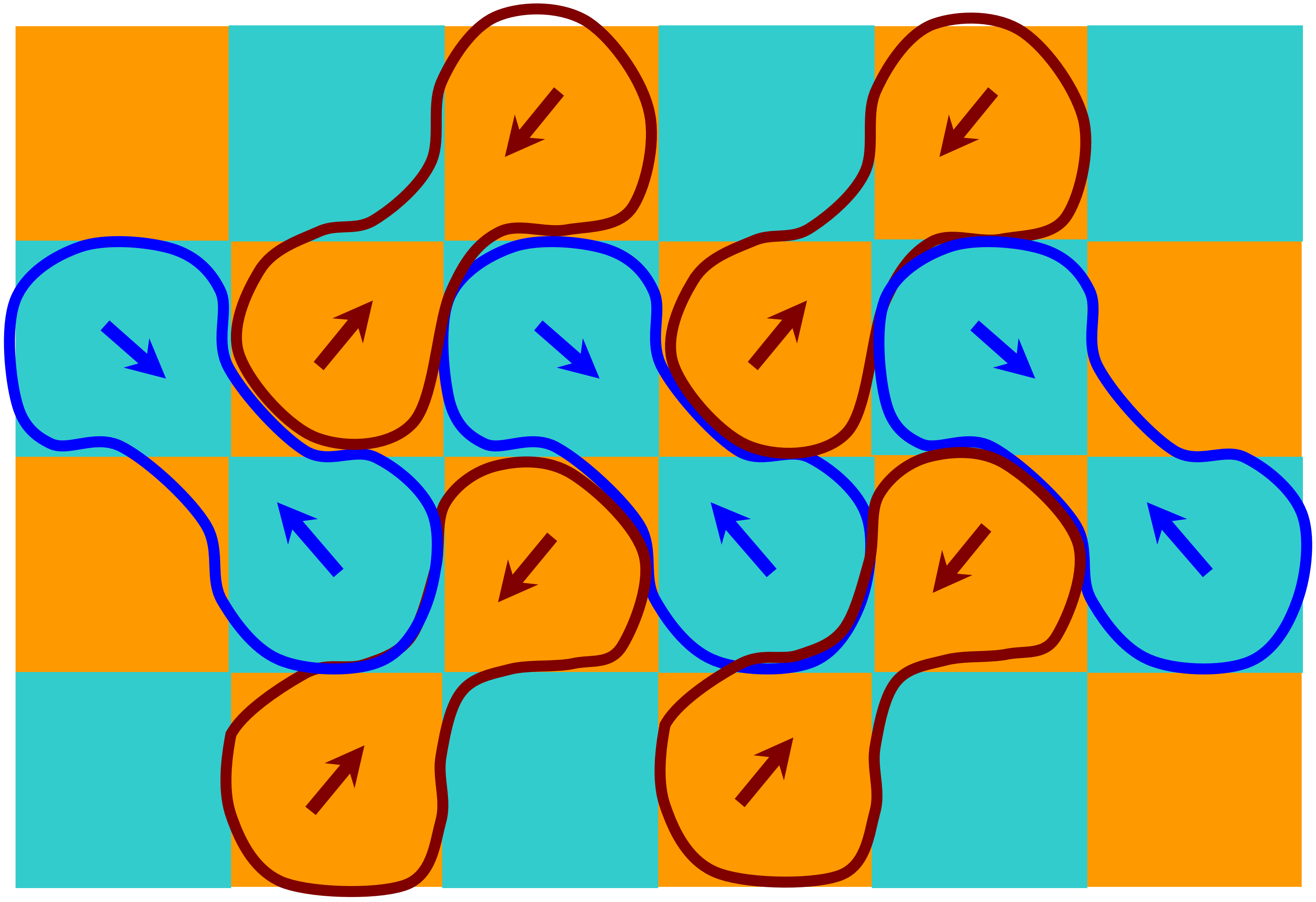
Decreasing of number of fingers

Fingers thicken with time. Thus the number of fingers should decrease. How it is possible?

Fusion of fingers (from above)

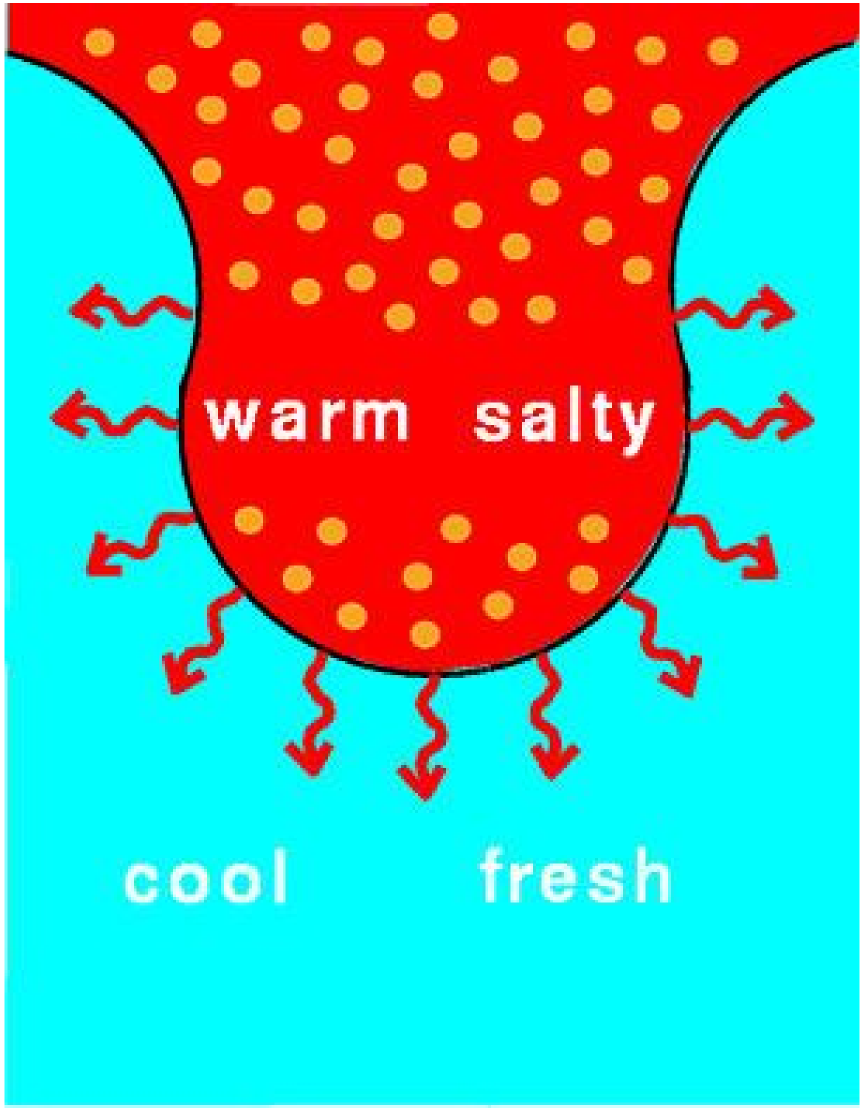


Fusion of fingers (from above)

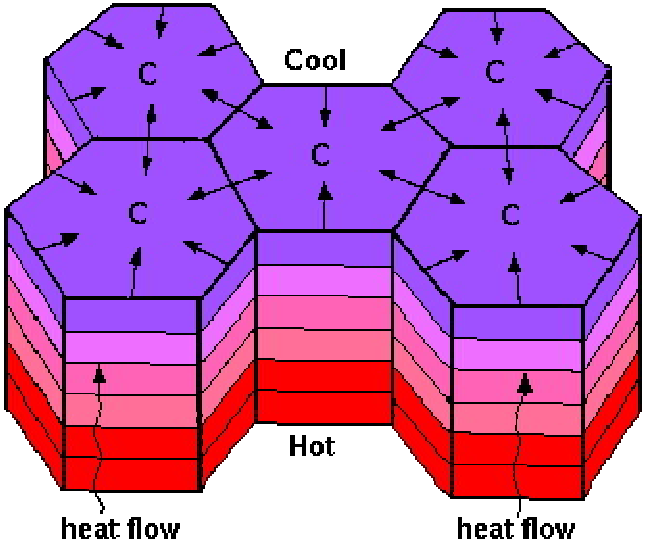


Another examples of finger structure

Ocean salt fingers



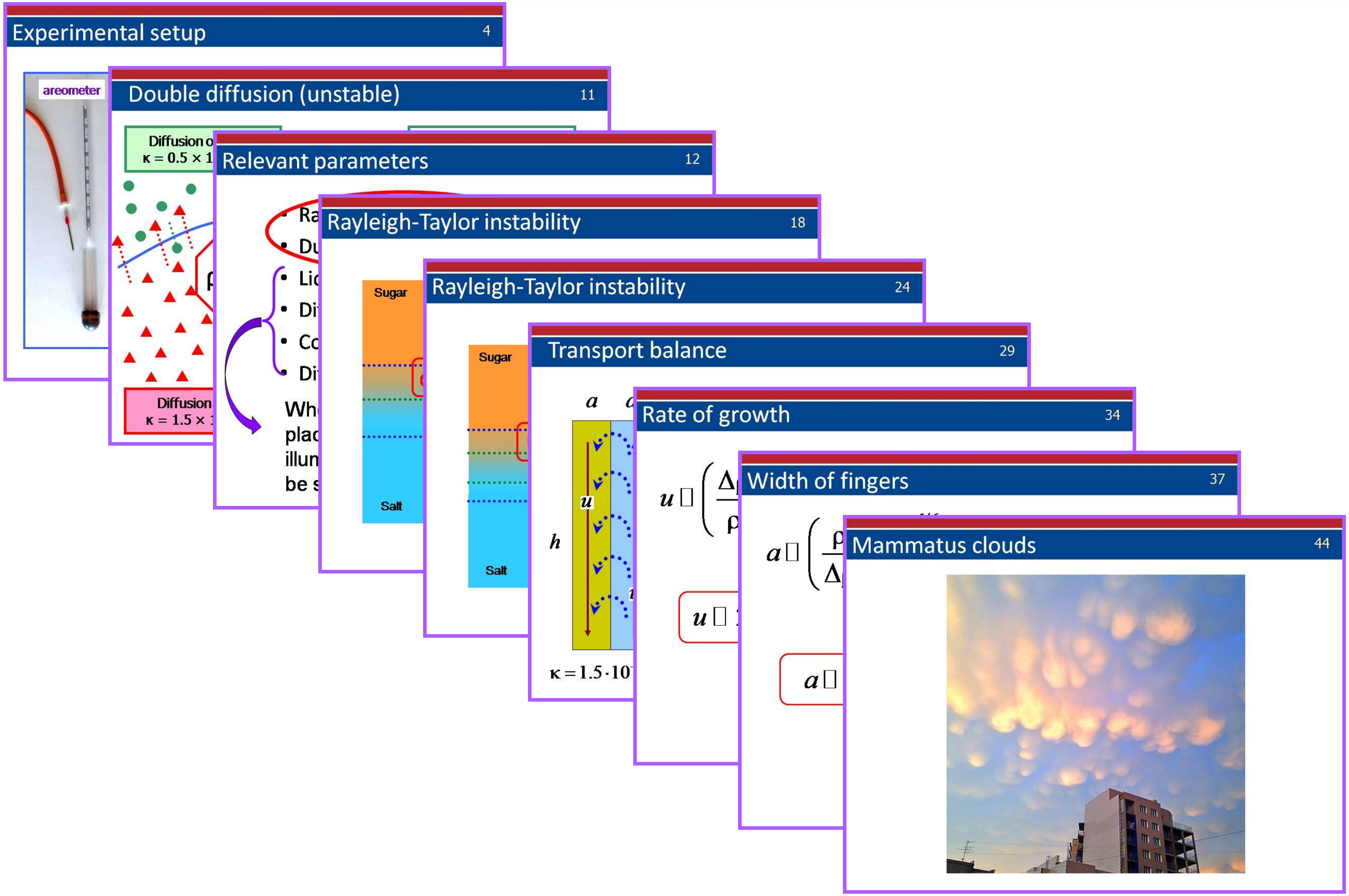
Columnar jointing basalts



Mammatus clouds

Summary

Main results



References

- Stern M.E., Turner J.S. (1969) “Salt fingers and convecting layers”. *Deep Sea Res.* **16**, 497–511.
- Shirtcliffe T.G.L., Turner J.S. (1970) “Observations of the cell structure of salt fingers”. *J. Fluid Mech.* **41**, 707–719.
- Linden P.F. (1973) “On the structure of salt fingers”. *Deep Sea Res.* **20**, 325–340.

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Thank you for your attention!